Decays of Four Intersecting Fluxbranes

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We consider decays of four intersecting fluxbranes which are obtained by considering a higher dimensional Kerr blackhole with four angular momentum parameters, which is the maximum number of angular momentum parameters in string/M-theory. As a result of the intersection, we get lower dimensional fluxbranes. Since generic magnetic fields break all supersymmetries, the resulting fluxbranes are unstable and will decay. Just as a single fluxbrane decays into the nucleation of spherical D6-branes; the intersecting ones decay into the nucleation of lower dimensional spherical branes. Contrary to a single fluxbrane case, the decay of four intersecting fluxbranes has additional decay channels. We also calculate the corresponding Euclidean action to obtain the decay rates. Although the action cannot be explicitly and simply written in terms of the magnetic parameters, we can extract some interesting results by taking various limits of the magnetic parameters.

I. INTRODUCTION

Obtaining an exact spectrum in nontrivial backgrounds is quite restricted in string theory. One example of such a nontrivial background is the Melvin background[1], which originally appeared as a solution of the Einstein-Maxwell system. There the magnetic field, which is gravitating, is aligned along one spatial direction in four spacetime dimensions. In string theory, there are two kinds of magnetic fields coming from NS-NS and R-R gauge fields and giving NS-NS Melvin background and R-R Melvin background, respectively. The R-R Melvin background is often called a fluxbrane. These Melvin backgrounds are related by a U-duality. Since the nontrivial magnetic fields are gravitating, the spacetime will be curved. Nevertheless, in the NS-NS Melvin background [2], one can calculate the exact string spectrum. However, in the fluxbrane background, the exact string spectrum is not yet available due to the nonperturbative effect of the dilaton.

Because the Killing spinors do not exist in these Melvin backgrounds the supersymmetry is generically broken. In that case, a closed string has tachyons as a ground state, which is a signal of instability. However, if one turns on two or more branes that intersect, there is a possibility of preserving partial supersymmetry.

As already has been discussed before in Ref.[3], the unstable fluxbrane in four dimensions decays into the nucleation of a pair creation of black holes. This is the magnetic analog of Schwinger pair production in an electric field. Later, this idea was generalized to general dimensional cases [4], and various decay modes were discussed. One of them was the creation of charged or uncharged spherical branes, including Witten's bubble of nothing[5]. In addition, intersecting fluxbranes were discussed, in which the lower-dimensional fluxbranes were obtained as a result. These lower-dimensional Melvin backgrounds can be obtained from Kaluza-Kline reduction from higher dimensions with a non-trivial identification of the circle. Another way to get a lower dimensional fluxbrane is to directly choose a metric ansatz and solve the Einstein equation [6]. Recently, this was discussed in relation to M-theory in a general case [2]. The technique of obtaining magnetic backgrounds in four dimensions was applied to M-theory[7]. By calculating the partition function, type IIA and type 0A strings were shown to be related by a shift of the magnetic field. This is an explicit example of the duality between type IIA theory and type 0A theory[8]. From a spin structure argument, one can see that the decays are different for type IIA and type 0A. In a type IIA string, the Melvin background decays into the nucleation of a pair of D6 and anti-D6 branes and for type 0A string, it decays into a bubble of nothing. More recently, two intersecting fluxbranes and their decays were considered in Ref.[9]. By numerical study the Euclidean action was shown to become infinite, and the fluxbranes were shown not to decay when the magnetic fields took on particular value. This is consistent with the existence of supersymmetry. In the presence of more intersecting branes, one can reduce along Killing vectors in more ways than as in the single-brane case. Thus, there are more decay modes and different kinds of brane creations corresponding to each Killing vector. Other properties of fluxbranes are discussed in Refs.[10, 11, 12, 13, 14].

In general, an Fp-brane in D-dimension has $ISO(p,1) \times SO(D-p-1)$ symmetry and a non-zero rank $(D-p-1)$ field strength, a dual field strength or a wedge product of field strengths tangent to the transverse dimensions. These Fp-branes couple to a D(p-1)-brane. When the fluxbranes intersect, the isometry reduces and results in lower-dimensional fluxbrane. In our work, we consider four intersecting fluxbranes. In particular, we will consider four intersecting seven-dimensional fluxbranes called F7-branes. This number of fluxbranes is the maximum possible number when we consider the positiveness of dimensions of the created branes. This maximum number of fluxbranes also comes if one considers the duality to the NS-NS Melvin background and

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its orbifold limit: by taking appropriate limits, we can obtain orbifolds like $\mathbb{C}^r/\mathbb{Z}_n$, and $r = 4$ is the maximum value for r in 10-dimensions [15]. If we consider five intersecting fluxbranes, we get a fluxbrane with a negative (-1) dimension, which has no interpretation in string theory. As can be expected, as we increase the number of intersecting fluxbranes, there are more ways of reduction corresponding to each Killing vector. Thus, we can see a number of decay modes as a result. We can check the existence of the supersymmetry. In an indirect way, we can construct a partition function for the general Melvin background (we have to consider the exactly solvable NS-NS Melvin background which is U-dual to a fluxbrane). Then, this partition function will vanish when the magnetic fields satisfy a particular relation. As an example, for four magnetic fields, the partition function [16] vanishes when the magnetic fields satisfy $B_1 = \pm B_2 \pm B_3 \pm B_4$. When the magnetic fields do not satisfy this relation, the closed string tachyonic modes are present [17]. For the discussion of the rolling string tachyon, see for example Ref.[18]. Another way is to directly solve the Killing spinor equation. The existence of the solution of the Killing spinor equation is known to be consistent with a vanishing partition function [19].

The rest of the paper is as follows: In Section II, we briefly review the Kaluza-Klein magnetic flux tube background and its decay. In Section III, we will consider four intersecting fluxbranes and their decay. With the intersecting fluxbranes, we can consider more Killing vectors and corresponding decay channels. We also calculate the Euclidean action. Since we cannot obtain the action as an explicit function of magnetic fields, it is not easy to analyze the result. However, when we take limits, it is much easier to analyze it. In Section IV, we conclude with some discussions.

II. KALUZA-KLEIN MAGNETIC FLUX TUBE BACKGROUND AND ITS DECAY

In Ref.[5], Witten pointed out that the original Kaluza-Klein vacuum $\mathrm{M}_4 \times S^1$ is semiclassically unstable and decays into a bubble of nothing. He started with a five-dimensional Euclidean flat space, where one direction was periodically identified on a circle. The fact that this space is unstable can be checked directly by considering fluctuations around the background, and exponentially growing modes are found, i.e., the existence of a (gravitational) instanton (or a bounce) solution. In five dimensions, the Euclidean Schwarzschild black hole solution acts as the bounce solution. Therefore, one can insist that the Kaluza-Klein vacuum decay through an instanton into the bubble of nothing. One can extend this idea to many other cases that have an instanton solution. One trivial extension is to regard any D-dimensional Euclidean Schwarzschild black hole as the bounce solution.

Now we can generalize Witten's idea to the case with a magnetic field. For that purpose, we need to consider the Melvin background. The four dimensional Melvin universe [1] can be obtained by dimensional reduction from five dimensions,

$$ds^2 = -dt^2 + dz^2 + d\rho^2 + \rho^2 d\phi^2 + (dx_5)^2, \qquad (1)$$

with nontrivial identification

$$(t, z, \rho, \phi, x_5) \equiv (t, z, \rho, \phi + 2\pi n_1 RB + 2\pi n_2, x_5 + 2\pi n_1 R), \qquad (2)$$

where $n_1, n_2 \in \mathbb{Z}$ and B is the magnetic field. In this case, since we have to twist both the ϕ and the x_5 directions, the Killing vector is given by

$$K = \frac{\partial}{\partial x_5} + B\frac{\partial}{\partial \phi}. \qquad (3)$$

The four (from $D = 5$) reduced dimensions can be read by using the general Kaluza-Klein ansatz

$$\begin{aligned} ds_D^2 \; &= \; \exp\left(\frac{4\phi}{\sqrt{D-2}}\right)\left(dx^D + 2A_\mu dx^\mu\right)^2 \\ &+ \; \exp\left(-\frac{4\phi}{(D-3)\sqrt{D-2}}\right) g_{E,\mu\nu} dx^\mu dx^\nu, \end{aligned} \qquad (4)$$

from which we can see that the action

$$S = \frac{1}{16\pi G_D}\int d^D x \sqrt{-g_D} R(g_D) \qquad (5)$$

reduces to

$$\begin{aligned} S \; &= \; \frac{1}{16\pi G_{D-1}}\int d^{D-1}x\sqrt{-g}\,\Big[R(g_{D-1}) \\ &- \; \frac{4}{D-3}(\nabla\phi)^2 - \exp\left(-\frac{4\sqrt{D-2}}{D-3}\phi\right)F^2\Big], \end{aligned} \qquad (6)$$

where $2\pi R G_{D-1} = G_D$. This action contains the dilaton ϕ and the gauge field strength $F_{\mu\nu}$, as well as the metric $g_{\mu\nu}$. The static Melvin solution to this action was studied in detail in Ref.[3]. It is described by a dilatonic C-metric or Ernst metric [20], where two blackholes are accelerating in opposite directions in a magnetic background.

In order to get the Melvin metric in four dimensions from five dimensions, we introduce $\tilde{\phi} = \phi - Bx_5$ in Eq. (1) in doing the KK reduction with the identification Eq. (2). Reducing along the Killing vector $\partial/\partial x_5$, we get the following solutions for the metric and the fields:

$$\begin{aligned} ds_4^2 &= \sqrt{\Lambda}\left[-dt^2 + d\rho^2 + dz^2\right] + \frac{1}{\sqrt{\Lambda}}\rho^2 d\tilde{\phi}^2, \\ \exp\left(-\frac{4\phi}{\sqrt{3}}\right) &= \Lambda = 1 + B^2\rho^2, \quad A_{\tilde{\phi}} = \frac{B\rho^2}{2\Lambda}. \end{aligned} \qquad (7)$$

As we can see, this solution describes a magnetic flux tube which is aligned in the z direction and is transverse to the (ρ, ϕ) plane. We can identify it as a flux one brane or F1-brane. In Ref.[4], the decay to the bubble of nothing (obtained by reducing along $\partial/\partial x_5$) or the creation of black hole pair accelerating in a magnetic background (obtained by reducing along $\partial/\partial x_5 + \partial/\partial\phi$) was discussed. One noticeable point is the fact that the solution describing the nucleation of two oppositely charged black holes coming from the Kerr instanton is the Ernst metric solution. For the creation of a pair of black holes (as can be compared to a KK monopole) we need a Kerr instanton, which we will discuss below. We can embed these into string theory or M-theory.

Melvin background in M-theory: A Review

In the rest of this section, we consider the Kaluza-Klein magnetic flux tube background in eleven-dimensions. Upon reduction to ten-dimensions, we get the four-dimensional analog of Melvin background. Let us begin with the 11-dimensional metric in cylindrical coordinates:

$$ds^2 = -dt^2 + dy_m dy^m + dx^2 + d\rho^2 + \rho^2 d\phi^2 + dz^2, \quad (8)$$

where $m = 1, \cdots, 6$. Now, let us take the following identifications:

$$\begin{aligned}&(t, y_m, x, \rho, \phi, z)\\ &\equiv (t, y_m, x, \rho, \phi + 2\pi n_1 RB + 2\pi n_2, z + 2\pi n_1 R), \quad (9)\end{aligned}$$

where n_1 and n_2 are integers. If we reduce along the Killing vector

$$K = \frac{\partial}{\partial z} + B\frac{\partial}{\partial\phi}, \quad (10)$$

we get the Melvin background. It is convenient to introduce the coordinate $\tilde{\phi} = \phi - Bz$. Using the relation between the 11-dimensional M-theory metric and the 10-dimensional string theory metric

$$ds_{11}^2 = e^{-\frac{2\phi}{3}} ds_{10}^2 + e^{\frac{4\phi}{3}} \left(dz + 2A_\mu dx^\mu\right)^2, \quad (11)$$

we get

$$\begin{aligned}ds_{10}^2 &= \sqrt{\Lambda}(-dt^2 + dy_m dy^m + dx^2 + d\rho^2) + \frac{1}{\sqrt{\Lambda}}\rho^2 d\tilde{\phi}^2,\\ e^{\frac{4\phi}{3}} &= \Lambda \equiv 1 + \rho^2 B^2, \quad A_{\tilde{\phi}} = \frac{B\rho^2}{2\Lambda}. \quad (12)\end{aligned}$$

This is an extension of the F1 brane of Eq. (7) and is called a seven-dimensional fluxbrane or F7 brane. Here, B is calculated from

$$B^2 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu}|_{\rho=0}. \quad (13)$$

The condition $|B| \ll 1/R$ is necessary in order to have a reliable perturbative string theory (small R) and a Kaluza-Klein ansatz ($\rho \gg R$). If only bosons are considered, the magnetic field B lies within the range

$$-\frac{1}{2R} < B \le \frac{1}{2R}. \quad (14)$$

When fermions are included, in order not to change the boundary condition (periodic boundary), B lies within the following range:

$$-\frac{1}{R} < B \le \frac{1}{R}. \quad (15)$$

Now, let us see the decay of the Kaluza-Klein magnetic background. As mentioned above, the instanton we are interested in will be given by the 11-dimensional Kerr black hole [21]:

$$\begin{aligned}ds^2 &= dy_m dy^m + dz^2 + \sin^2\theta(r^2 - \alpha^2)d\phi^2\\ &- \frac{\mu}{\rho^2}(dz + \alpha\sin^2\theta d\phi)^2 + \frac{\rho^2}{r^2 - \alpha^2 - \mu}dr^2\\ &+ \rho^2 d\theta^2 + r^2\cos^2\theta d\psi^2, \quad (16)\end{aligned}$$

where $\rho^2 = r^2 - \alpha^2\cos^2\theta$. The apparent singularity is at $r^2 = r_H^2 \equiv \mu + \alpha^2$. We express the parameters of this background as

$$B = \frac{\alpha}{\mu}, \quad R = \frac{1}{\kappa} = \frac{\mu}{\sqrt{\mu + \alpha^2}}, \quad (17)$$

where $\omega \equiv i\Omega = iB$ is a Lorentzian angular velocity and κ is a surface gravity. If we set α to zero the metric becomes a flat 11-dimensional Euclidean space. We can dimensionally reduce it in two ways: either $K = \partial/\partial z + \Omega\partial/\partial\phi$ or $K' = \partial/\partial z + (B \pm (1/R))\partial/\partial z$, which have zero norm at $r = r_H$. The reduction can be done by redefining $\tilde{\phi} = \phi - Bz$ or $\tilde{\phi} = \phi - (B \pm (1/R))z$. The latter case is called the shifted instanton. The conical singularity at $r = r_H$ can be removed if z has a period of $2\pi R$.

We can estimate the decay rate as

$$\Gamma \sim e^{-I_E}, \quad (18)$$

where I_E is the Euclidean action,

$$I_E = -\frac{1}{16\pi G_{11}}\int d^{11}x\sqrt{g}R - \frac{1}{8\pi G_{11}}\int d^{10}x\sqrt{h}(\mathcal{K} - \mathcal{K}_0). \quad (19)$$

Here $\mathcal{K}$ is the trace of the extrinsic curvature of the boundary, and $\mathcal{K}_0$ is the same quantity for the Melvin background. For unshifted Kerr instanton, we have

$$I_E = \left(\frac{\pi V_6}{8G_{10}}\right)\frac{R^2}{1 - (BR)^2}, \quad (20)$$

while for a shifted instanton, we have

$$I_E = \left(\frac{\pi V_6}{8G_{10}}\right)\frac{R^2}{1 - (1 - |B|R)^2}. \quad (21)$$

In the above, V_6 is the volume of a unit 6-sphere.

Since the Kerr instanton has a topology of $\mathbb{R}^2 \times S^3$, it admits a single-spin structure. After a parallel transport around a closed curve at infinity, the spin picks up a phase of $-e^{\pi R\alpha\Gamma/\mu}$ for both the shifted and the unshifted instanton. On the other hand, the magnetic background has a topology of a circle (since $M^4 \times S^1$) and thus there can be two types of spin structures:

$$e^{\pi RB\Gamma}, \quad \text{and} \quad -e^{\pi RB\Gamma}, \tag{22}$$

where Γ is an element of the Lie algebra of Spin(1,10) satisfying $\Gamma^2 = -1$. If we choose the first spin structure corresponding to type IIA, the appropriate Kerr instanton is the shifted one, $-e^{\pi R(B\mp 1/R)\Gamma} = +e^{\pi RB\Gamma}$ ($\alpha/\mu = B \pm (1/R)$), whose Killing vector is $K' = K \mp \frac{1}{R}\frac{\partial}{\partial\phi}$. We choose the positive sign for a positive B field while we choose a negative sign for negative B field. The second choice picks an unshifted instanton as the decay mode for type 0A, whose Killing vector is $K = \frac{\partial}{\partial z} + B\frac{\partial}{\partial\phi}$. The instanton of type 0A decays to Witten's bubble of nothing, and that of type IIA decays to pair creation of 6-branes.

III. FOUR INTERSECTING FLUXBRANES

The fixed point sets under the Killing vectors isometry correspond to branes or decay products after reduction. We will apply the results of Ref.[4] to our case.

In order to discuss the decay of four intersecting fluxbranes, we have to consider the 11-dimensional Kerr black hole [21] with four angular momentum parameters for each transverse $\mathbb{R}^2$. Before studying that, let us briefly review intersecting fluxbranes as discussed in Ref.[4]. Consider 11-dimensional metric

$$ds_{11}^2 = \sum_{i=1}^{5}(d\rho_i^2 + \rho_i^2 d\phi_i^2) + dy^2. \tag{23}$$

Since a fluxbrane, for example, F1 in four dimensions, can be considered as a magnetic field piercing through two transverse plane, intersecting fluxbranes in higher dimensions can be obtained by taking two orthogonal planes and turning on a magnetic flux on each plane. This can be done by the following: We reduce along the Killing direction with the Killing field

$$K = \frac{\partial}{\partial y} + \sum B_i \frac{\partial}{\partial \phi_i}. \tag{24}$$

If $\tilde{\phi}_i = \phi_i - B_i y$ is introduced, the new Killing field will be $K' = \partial/\partial y$. We rewrite the metric as

$$\begin{aligned} ds^2 &= \Lambda\left[dy + \Lambda^{-1}\sum_i B_i\rho_i^2 d\tilde{\phi}_i\right]^2 + \sum_i(d\rho_i^2 + \rho_i^2 d\tilde{\phi}_i{}^2) \\ &- \Lambda^{-1}\left(\sum_j B_j\rho_j^2 d\tilde{\phi}_i{}^2\right)^2, \end{aligned} \tag{25}$$

where

$$\Lambda = 1 + \sum_i B_i^2 \rho_i^2. \tag{26}$$

Then, the reduced 10-dimensional metric will be

$$\begin{aligned} ds_{10}^2 &= \Lambda^{\frac{1}{8}}\left[-dt^2 + \sum_i(d\rho_i^2 + \rho_i^2 d\tilde{\phi}_i{}^2)\right. \\ &= \left. -\Lambda^{-1}\left(\sum_j B_j\rho_j^2 d\tilde{\phi}^2\right)\right] \\ \Lambda &= e^{-\frac{4}{\sqrt{7}}\phi}, \quad A = \frac{1}{2\Lambda}\sum_I B_i\rho_i^2 d\tilde{\phi}_i. \end{aligned} \tag{27}$$

The case of a F7-brane arises when only one direction is chosen. In this case, the F7-brane in 11-dimension has $ISO(7,1)\times SO(3)$ symmetry and a non-zero-rank three field strength, a dual field strength, or a wedge product of field strengths tangent to the transverse dimensions. From the above general result, the extension to four B_i nonzero, which is our main consideration here, by reducing the Poincare symmetry with the choice of the four $\tilde{\phi}_i$, is easy. As a result of turning on more magnetic parameters, the dimensionality of the resulting fluxbrane is reduced.

Now, let us discuss the decay of four intersecting fluxbranes. The 11-dimensional metric we are considering with four angular-momentum parameters is given by [21]

$$\begin{aligned} ds_{11}^2 &= dz^2 + \sum_{i=1}^{4}(r^2-\alpha_i^2)(d\mu_i^2 + \mu_i^2 d\phi_i^2) \\ &+ r^2(d\mu_5^2 + \mu_5^2 d\phi_5^2) + \frac{\Pi F}{\Pi - \mu r^2}dr^2 \\ &- \frac{\mu r^2}{\Pi F}(dz^2 + \sum_{i=1}^{4}\alpha_i\mu_i^2 d\phi_i)^2, \end{aligned} \tag{28}$$

where

$$\Pi = r^2\prod_{i=1}^{4}(r^2-\alpha_i^2), \qquad F = 1 + \sum_{i=1}^{4}\frac{\alpha_i^2\mu_i^2}{r^2-\alpha_i^2}, \tag{29}$$

and

$$\mu = \prod_{i=1}^{4}(r_H^2 - \alpha_i^2), \tag{30}$$

with the constraint of

$$\mu_5^2 = 1 - \sum_{i=1}^{4}\mu_i^2. \tag{31}$$

To avoid a conical singularity at $r = r_H$, we must identify the coordinates as

$$\begin{aligned} &z \equiv z + 2\pi R, \\ &\phi_i \equiv \phi_i + 2\pi\Omega_i R + 2\pi m_i, \quad (i = 1,2,3,4), \end{aligned} \tag{32}$$

where

$$R = \frac{2\mu r_H^2}{\Pi'(r_H) - 2\mu r_H} = \mu\frac{1}{\Sigma} \tag{33}$$

and

$$\begin{aligned}\Sigma &= r_H[4r_H^6 - 3r_H^4(\alpha_1^2+\alpha_2^2+\alpha_3^2+\alpha_4^2) \\ &+ 2r_H^2((\alpha_1^2+\alpha_2^2)(\alpha_3^2+\alpha_4^2)+\alpha_1^2\alpha_2^2+\alpha_3^2\alpha_4^2)) \\ &- (\alpha_1^2+\alpha_2^2)\alpha_3^2\alpha_4^2 - (\alpha_3^2+\alpha_4^2)\alpha_1^2\alpha_2^2] \end{aligned} \tag{34}$$

which can be rewritten as

$$r_H[(r_H^2-\alpha_1^2)(r_H^2-\alpha_2^2)(r_H^2-\alpha_3^2)+(2,3,4)+(1,3,4)+(1,2,4)], \tag{35}$$

with (i,j,k) meaning $(r_H^2-\alpha_i^2)(r_H^2-\alpha_j^2)(r_H^2-\alpha_k^2)$ in the brackets. In the asymptotic limit, the solution goes to flat spacetime with non-trivial identification. In this region, the solution looks like the Euclidean 11-dimensional intersecting fluxbrane (F1) solution. This causes us to identify the magnetic fields as

$$B_i = \Omega_i \quad \text{for } i = 1,2,3,4. \tag{36}$$

A. Spin Structure and Decay of Fluxbranes

As we saw in the previous section, the Killing vector is important if we want to consider the decays of the fluxbrane. It determines whether the result is type IIA or type 0A and the types of branes created. The types of nucleation depend on the fixed point sets of the Killing vector.

Now we briefly see what the fixed point sets are and how they are related to the decay product by looking into the examples summarized in Ref.[4] and references therein. Consider the isometry generated by the Killing vector K. For the associated Killing field f_a, we define

$$f_{ab} \equiv f_{a;b}. \tag{37}$$

At a fixed point, $f_a = 0$. In addition, let the kernel of f_{ab} have dimension p. The vectors in the kernel are directions in the tangent space at the fixed point and are invariant under the action of the symmetry. The p-dimensional subspace of fixed points is what will be nucleated after decay of fluxbranes. In four dimensions, $p=0$ is called a 'nut', and $p=2$ is called a 'bolt'. The simple example is a five-dimensional Euclidean Schwarzschild solution:

$$\begin{aligned} ds^2 &= dz^2 + \left(1-\frac{2m}{r}\right)d\tau^2 + \left(1-\frac{2m}{r}\right)^{-1}dr^2 \\ &+ r^2(d\theta^2+\sin^2\theta d\phi^2), \end{aligned} \tag{38}$$

where τ has period $2\pi R$ with $R=4m$. If we reduce along the Killing vector $\partial/\partial\tau$, we find a minimum of two-sphere or bolt as the fixed point set of that circle action. Consider now the alternative Killing vector $\partial/\partial\tau + (1/R)\partial/\partial\phi$. This has fixed points at the north and the south poles, being nut($\theta=0$) and anti-nut($\theta=\pi$), respectively. We apply this idea to following decays:

In our case of four intersecting fluxbranes, various kinds of Killing vectors and the corresponding decays can be considered. Now let us consider them one by one.

- Reduction along the Killing vector

$$K_1 = \frac{\partial}{\partial z} + \sum_{i=1}^{4}\Omega_i\frac{\partial}{\partial\phi_i}. \tag{39}$$

The result will give $B_i = \Omega_i$ $(i=1,\cdots 4)$ and the fixed-point set is a 9-sphere (entire horizon), which is just Witten's bubble of nothing. From a consideration of the spin structure, this results in a type-0A theory.

- Reduction along the Killing vector

$$K_2 = K_1 \mp \frac{1}{R}\frac{\partial}{\partial\phi_i}. \tag{40}$$

The result will give magnetic fields $B_i = \Omega_i \mp \frac{1}{R}$ and $B_{j\neq i} = \Omega_{j\neq i}$. There are four (${}_4\mathrm{C}_1$) types of decays, and the fixed point set is a charged spherical 6-brane. From a consideration of the spin structure, this results in a type IIA theory.

- Reduction along the Killing vector

$$K_3 = K_1 \mp \frac{1}{R}\frac{\partial}{\partial\phi_i} \mp \frac{1}{R}\frac{\partial}{\partial\phi_j}, \quad i\neq j. \tag{41}$$

There are six types (${}_4\mathrm{C}_2$) of decays. The result will give magnetic fields $B_i = \Omega_i \mp \frac{1}{R}$ and $B_j = \Omega_j \mp \frac{1}{R}$, and the fixed point sets are uncharged spherical 4-branes. From a consideration of the spin structure, this results in type 0A theory.

- Reduction along the Killing vector

$$K_4 = K_1 \mp \frac{1}{R}\frac{\partial}{\partial\phi_i} \mp \frac{1}{R}\frac{\partial}{\partial\phi_j} \mp \frac{1}{R}\frac{\partial}{\partial\phi_k}, \quad i\neq j\neq k. \tag{42}$$

There are four (${}_4\mathrm{C}_3$) types of decays. The result will give magnetic fields $B_{i,j,k} = \Omega_{i,j,k} \mp \frac{1}{R}$, and the fixed point set is a uncharged spherical 2-brane. From the spin structure, this results in a type IIA theory.

- Reduction along the Killing vector

$$K_5 = K_1 \mp \frac{1}{R}\sum_{i=1}^{4}\frac{\partial}{\partial\phi_i}. \tag{43}$$

There is a single (${}_4\mathrm{C}_4$) type of decay. The result will give magnetic fields $B_i = \Omega_i \mp \frac{1}{R}$, and the fixed point set is an uncharged 0-brane. From a consideration of the spin structure, this results in a type 0A theory.

The first, the second, and the fifth cases are of the type that were considered in Ref.[9]. The third and the fourth cases are new ones, which are due to the intersection of more than two fluxbranes.

B. Calculation of the Action

The decay rate of the fluxbranes is equal to the creation rate of the spherical D-branes and is given by e^{-I_E}. To see this explicitly, we have to calculate the Euclidean action. For this calculation, we apply the method introduced in Ref.[4]. The instanton we are considering is Ricci flat, and the contribution to the action is zero. Thus the only non-vanishing action comes from the boundary term

$$I_{E,11} = -\frac{1}{8\pi G_{11}} \int_{r\to\infty} d^{10}x\sqrt{h}(\mathcal{K} - \mathcal{K}_0), \quad (44)$$

where h is the determinant of the induced metric on a constant r surface, $\mathcal{K}$ is the extrinsic curvature of this metric, and $\mathcal{K}_0$ is the extrinsic curvature of the reference background (in our case $\mu = 0$).

The induced metric $r = 0$ can be written as follows:

$$ds_{10}^2 = dz^2 + \sum_{i=1}^{4}(r^2 - \alpha_i^2)(d\mu_i^2 + \mu_i^2 d\phi_i^2)$$
$$+r^2(d\mu_5^2 + \mu_5^2 d\phi_5^2) - \frac{\mu r^2}{\Pi F}(dz + \sum_{i=1}^{4}\alpha_i\mu_i^2 d\phi_i)^2, \quad (45)$$

where we have to put

$$\mu_5^2 = 1 - \sum_{i=1}^{4}\mu_i^2, \quad d\mu_5^2 = (\sum_{i=1}^{4}\mu_i d\mu_i)^2 \quad (46)$$

in the metric.

We will use the relation [4]

$$\sqrt{h}\mathcal{K} = \hat{n}\sqrt{h}, \quad (47)$$

where the unit normal vector $\hat{n}$ is given by

$$\hat{n} = \sqrt{\frac{\Pi - \mu r^2}{\Pi F}}\frac{\partial}{\partial r}. \quad (48)$$

From the given metric, we can get the determinant $\sqrt{h}$, which is given as follows:

$$\sqrt{h} = (\prod_{i=1}^{4}\mu_i)\sqrt{(r^8 + Dr^6 - Cr^4 + Br^2 - A)(\Pi - \mu r^2)}, \quad (49)$$

where

$$\begin{aligned} A &= \left(\prod_{i=1}^{4}\alpha_i^2\right)\left((\sum_{j=1}^{4}\mu_i^2) - 1\right), \\ B &= \left(\prod_{i=1}^{4}\alpha_i^2\right)\sum_{j=1}^{4}\left(\frac{(\sum_{k=1}^{4}\mu_k^2) - \mu_j^2 - 1}{\alpha_j^2}\right), \\ C &= \sum_{i,j=1,i<j}^{4}\alpha_i^2\alpha_j^2(\mu_i^2 + \mu_j^2 - 1), \\ D &= \sum_{i=1}^{4}\alpha_i^2(\mu_i^2 - 1). \end{aligned} \quad (50)$$

Notice that for the case of two intersecting fluxbranes ($\alpha_3 = \alpha_4 = 0$), our result reduces to the one in Ref.[9], i.e.,

$$\begin{aligned} A &= 0,\ B = \alpha_1,\ C = \alpha_1^2\alpha_2^2(\mu_1^2 + \mu_2^2 - 1), \\ D &= \alpha_1^2(\mu_1^2 - 1) + \alpha_2^2(\mu_2^2 - 1). \end{aligned} \quad (51)$$

We can evaluate $\sqrt{h}\mathcal{K} = \hat{n}\sqrt{h}$ as

$$\begin{aligned} \hat{n}\sqrt{h} &= \frac{\mu_1\mu_2\mu_3\mu_4}{2} \\ &\times [\frac{(\Pi - \mu r^2)(8r^7 + 6Dr^5 - 4Cr^3 + 2Br)}{\sqrt{r^8 + Dr^6 - Cr^4 + Br^2 - A}} \\ &+ \frac{(r^8 + Dr^6 - Cr^4 + Br^2 - A)(\Pi' - 2\mu r)}{\sqrt{r^8 + Dr^6 - Cr^4 + Br^2 - A}}]. \end{aligned} \quad (52)$$

Since we can take the limits

$$\lim_{r\to\infty}\frac{\sqrt{h}}{\sqrt{h}|_{\mu=0}} = 1 - \frac{1}{2}\frac{\mu}{r^8}, \qquad \mathcal{K} = \frac{\hat{n}\sqrt{h}}{\sqrt{h}}, \quad (53)$$

we can obtain the integrand in the boundary term as

$$\begin{aligned} &\lim_{r\to\infty}\sqrt{h}(\mathcal{K} - \mathcal{K}_0) \\ &= \lim_{r\to\infty}\left(\hat{n}\sqrt{h} - \hat{n}\sqrt{h}|_{\mu=0} + \frac{1}{2}\frac{\mu}{r^8}\hat{n}\sqrt{h}|_{\mu=0}\right) \\ &= \lim_{r\to\infty}\left(\frac{\partial}{\partial\mu}\hat{n}\sqrt{h} + \frac{1}{2}\frac{1}{r^8}\hat{n}\sqrt{h}|_{\mu=0}\right)\mu. \end{aligned} \quad (54)$$

This becomes

$$\begin{aligned} \lim_{r\to\infty}\frac{\mu_1\mu_2\mu_3\mu_4}{2}\mu &\left[\frac{-r^2(8r^7 + 6Dr^5 - 4Cr^3 + 2Br) + (r^r + Dr^6 - Cr^4 + Br^2 - A)(-2r)}{\sqrt{\Pi F(r^8 + Dr^6 - Cr^4 + Br^2 - A)}}\right. \\ &\left. + \frac{1}{2}\frac{1}{r^8}\frac{\Pi(8r^7 + 6Dr^5 - 4Cr^3 + 2Br) + (r^8 + Dr^6 - Cr^4 + Br^2 - A)\Pi'}{\sqrt{\Pi F(r^8 + Dr^6 - Cr^4 + Br^2 - A)}}\right]. \end{aligned} \quad (55)$$

Surprisingly, the final result is simplified to

$$\lim_{r\to\infty}\sqrt{h}(\mathcal{K}-\mathcal{K}_0) = -\frac{\mu_1\mu_2\mu_3\mu_4}{2}\mu. \tag{56}$$

This final result looks exactly like the result obtained in Ref. [9] but it is, in fact, different in that our μ contains four magnetic parameters instead of two. Our result is the most general one in that in ten-dimensional string theory or eleven dimensional M-theory, the upper limit on the number of intersecting fluxbrane is four;furthermore, by turning off the magnetic parameters, we get results that reduce to lower-dimensional, known decays of two intersecting fluxbranes or a single fluxbrane.

Now, we are ready to calculate the action. After putting all these into the action, we get

$$I_{10,E} = \frac{V_9}{16\pi G_{10}}\mu, \tag{57}$$

where we define

$$V_9 = \int_0^{2\pi} d\phi_1\cdots d\phi_5 \int d\mu_1\cdots d\mu_4\ \mu_1\cdots\mu_4 \tag{58}$$

as the volume of a unit 9-sphere and we have used $G_{10} = 2\pi R G_9$. Now let us analyze the action by taking particular limits. As in Eq. (31), α_i's, R and μ have complicated relations and it is not easy to directly see the behavior of the action just as for the single-brane case. It is not possible to express the action as magnetic fields as before. Hence, in order to read some physics we discuss some particular limits. As done in Ref. [7] or [9], if one can figure out the behavior or diagram (say for shifted or unshifted) of the action, one can draw or figure out the action by shifting the diagram or the values of the action corresponding to each Killing vector. In our case, we start first with a Killing vector without shifting and continue to other Killing vectors with increasing number of shifts.

First, consider the case without a shifting in the magnetic parameters, say the unshifted instanton. Note that the angular velocities Ω_i's are given by

$$\begin{aligned}\Omega_i &= \frac{\alpha_i}{r_H^2-\alpha_i^2}\\ &= \frac{\alpha_i(r_H^2-\alpha_j^2)(r_H^2-\alpha_k^2)(r_H^2-\alpha_l^2)}{\mu},\\ &\text{where } i\neq j\neq k\neq l.\end{aligned} \tag{59}$$

Multiplying Eq.(33) by R gives

$$\begin{aligned}\Omega_i R &= \frac{\alpha_i\mu}{(r_H^2-\alpha_i^2)\Sigma}\\ &= \frac{\alpha_i(r_H^2-\alpha_j^2)(r_H^2-\alpha_k^2)(r_H^2-\alpha_l^2)}{\Sigma},\\ &\text{where } i\neq j\neq k\neq l.\end{aligned} \tag{60}$$

Let us take the limits

$$\alpha_i\to\pm\infty, \qquad \alpha_{j\neq i} = \text{constant} \ll \alpha_i \tag{61}$$

In this limit we have to take $r_H\to|\alpha_i|$ to keep R fixed (in this way only one of the terms survives in Eq.(35) among the four terms, and this cancels the denominator), and in order to keep the radius (R) fixed, take $\mu\to R|\alpha_i|^7$.

For example, take $i=1$, that is, consider $\alpha_1\to\pm\infty$. Then the other α's are too small to be negligible. To keep R fixed, take $r_H\to|\alpha_1|$ and $\mu\to R|\alpha_1|^7$. Then, three terms in Eq. (35) vanish, but the $(2,3,4)$ term does not. Therefore, $\Omega_1 R$ becomes ±1 in the above limit. This critical point is one of several critical points in which supersymmetry restores and generalizes the cases for only one magnetic parameter [4, 7] or two [9], depending the number of fluxbranes. We can follow the same procedure for the other α's. In this way, we find that the action diverges due to dependence of μ.

In summary, with the help of Eq. (35), by taking the limit $\alpha_i\to\infty$, we arrive at

$$\Omega_i R\to\pm1, \quad \Omega_{j\neq i}\to 0 \ \text{ and } \ \mu\to R|\alpha_i|^7. \tag{62}$$

Thus, in this limit, the Euclidean action I_E goes to infinity so that the decay rate vanishes, and no decay will happen. Since the parameter space spanned by the B_iR is 4-dimensional, it is difficult to plot the values of I_E as a function of B_iR's. However, we can easily see that the value of the action diverges at the following points: $(\pm1,0,0,0)$, $(0,\pm1,0,0)$, $(0,0,\pm1,0)$, and $(0,0,0,\pm1)$, where ±1 is a value of B_iR. The value of I_E decreases smoothly off the divergent peaks. This result corresponds to a type 0A theory, and there are eight points at which such a divergence occurs. Now, let us see the next decay mode with one shifted direction.

In this case, it is enough to shift the magnetic parameters $\Omega_iR\to1\pm\Omega_iR$ for each direction. The maximum point where the action diverges is now shifted. For instance, if we reduce along the 1-direction, then the above points are shifted to $(0,0,0,0)$, $(\pm1,\pm1,0,0)$, $(\pm1,0,\pm1,0)$, and $(\pm1,0,0,\pm1)$. This results in a type IIA theory. When we act further, one more shifting, we get the result for the third decay mode: For the reduction along the 1 and the 2 directions, the result we have peaks at the points $(0,\pm1,0,0)$, $(\pm1,0,0,0)$, $(\pm1,\pm1,\pm1,0)$, and $(\pm1,\pm1,0,\pm1)$, corresponding to a type 0A theory. For the reduction along the directions 1, 2, and 3 we have $(0,\pm1,\pm1,0)$, $(\pm1,0,\pm1,0)$, and $(\pm1,\pm1,0,0)$, $(\pm1,\pm1,\pm1,\pm1)$, corresponding to a type IIA theory. For the reduction along all directions we have $(0,\pm1,\pm1,\pm1)$, $(\pm1,0,\pm1,\pm1)$, $(\pm1,\pm1,0,\pm1)$, and $(\pm1,\pm1,\pm1,0)$, corresponding to a type 0A theory. We summarize this simple example in Table 1. (Remember that for Killing vector K_1 there is no shift in the parameters Ω_iR_i. For other Killing vectors, Ω_iR_i has to be shifted to $1\pm\Omega_iR_i$ for corresponding directions):

So far, we considered the diverging points by taking particular examples. The interesting case is when the reduction is done to an odd number of directions. These correspond to type IIA theories. When three directions are chosen, the maximum diverging action occurs when $B_1=\pm B_2=\pm B_3=\pm B_4$. This can also be represented

TABLE I: Divergent points of the Euclidean action $I_{E,11}$ for each Killing vectors

Killing Vector	Divergent points of I_{11}^E $(\Omega_1 R_1, \Omega_2 R_2, \Omega_3 R_3, \Omega_4 R_4)$			
K_1	$(\pm1,0,0,0)$	$(0,\pm1,0,0)$	$(0,0,\pm1,0)$	$(0,0,0,\pm1)$
K_2	$(0,0,0,0)$	$(\pm1,\pm1,0,0)$	$(\pm1,0,\pm1,0)$	$(\pm1,0,0,\pm1)$
K_3	$(0,\pm1,0,0)$	$(\pm1,0,0,0)$	$(\pm1,\pm1,\pm1,0)$	$(\pm1,\pm1,0,\pm1)$
K_4	$(0,\pm1,\pm1,0)$	$(\pm1,0,\pm1,0)$	$(\pm1,\pm1,0,0)$	$(\pm1,\pm1,\pm1,\pm1)$
K_5	$(0,\pm1,\pm1,\pm1)$	$(\pm1,0,\pm1,\pm1)$	$(\pm1,\pm1,0,\pm1)$	$(\pm1,\pm1,\pm1,0)$

by $B_1 = \pm B_2$ and $B_3 = \pm B_4$ or $B_1 = \pm B_2 \pm B_3 \pm B_4$. Of course this result contains the case of one reduced direction (second case of the above decay) which results in $B_i = \pm B_j (B_{\rm others} = 0)$. Our result is consistent with the result of the Killing spinor consideration[19].

IV. CONCLUSION AND DISCUSSION

We have considered four intersecting fluxbranes and investigated the decay modes by looking into the fixed point sets of the Killing vectors and the spin structures. There are more decay channels because there are more Killing vectors in the four-intersecting-fluxbrane case compared to the case of a single or two fluxbrane decay[9]. As a result, we obtained various kinds of branes, including the ones obtained in Ref. [9]. The new ones are spherical 4- and 2-branes and a 0-brane.

Because of the complicated relation between the magnetic field parameters and the parameters appearing in the action, we do not have an explicit relation of the action in terms of the magnetic field parameters. Since the decay rate of the fluxbranes should be equal to the creation rate of the branes, we can, in principle, calculate the action by calculating the DBI action in a type IIA case, and which will give the result for small magnetic fields. The calculation for the single-fluxbrane case was done in Ref.[7]. Extending such a calculation to an intersecting case would be difficult.

Formerly, the Kerr instanton was considered to get a pair creation of black holes in a Melvin background. In our case, we obtained a spherical brane, which is a fixed point set, as a result of decays and obtained a finite action that gave a finite decay rate, except for some diverging values. Note that in the case of F7-brane decay, a spherical 6-brane is created and can be interpreted as a D6-brane. However, in considering intersecting fluxbranes our procedure does not generate RR gauge fields that couple to some lower D-branes. It will be interesting to see their stability, i.e., to check whether the created spherical branes will expand, be stable, or shrink. As in Ref.[22] one can also consider a time-dependent background, such as a de Sitter background, as a result of decay from the Kerr instanton. One can further consider some other gravitational instantons and their decays. Other interesting cases are a consideration of the electric-magnetic duality of the same theory. In the current paper we considered and obtained magnetically charged cases. By duality, one can obtain electrically charged branes and the construction of fluxbranes that couple $\mathrm{D}p - \bar{\mathrm{D}}p$. It is well understood that the F7-brane plays a role in stabilizing the $\mathrm{D6} - \bar{\mathrm{D}}6$.

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